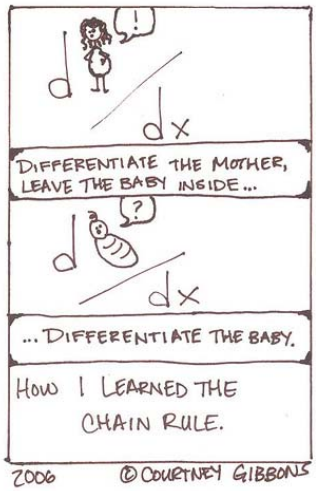
Advance KENTUCKY

AP Calculus Study Session Handout

Implicit Differentiation



Implicit Differentiation

The AP Calculus Exams include multiple-choice and free-response questions in which students are expected to differentiate implicitly to find the derivative. This includes applications of related rates which require students to be able to use implicit differentiation.

Questions involving implicit differentiation are usually on the no calculator portion of the AP Calculus Exams.

What Students Should Be Able to Do

- Differentiate an implicit relation.
- Verify the derivative of an implicit relation.
- Write an equation for a line tangent to the graph of an implicit relation at a particular point.
- Find the second derivative of an implicit relation.
- Find coordinates of points at which an implicit relation has horizontal and/or vertical tangent lines.
- Solve related rates problems.

Free Response Questions

2008 AB 6 Form B

Consider the closed curve in the *xy*-plane given by

$$x^2 + 2x + y^4 + 4y = 5.$$

(a) Show that $\frac{dy}{dx} = \frac{-(x+1)}{2(y^3+1)}$.

(b) Write an equation for the line tangent to the curve at the point (-2, 1).

(c) Find the coordinates of the two points on the curve where the line tangent to the curve is vertical.

(d) Is it possible for this curve to have a horizontal tangent at points where it intersects the *x*-axis? Explain your reasoning.

2005 AB 5 Form B

Consider the curve given by $y^2 = 2 + xy$.

(a) Show that $\frac{dy}{dx} = \frac{y}{2y-x}$.

(b) Find all points (x, y) on the curve where the line tangent to the curve has slope $\frac{1}{2}$.

(c) Show that there are no points (x, y) on the curve where the line tangent to the curve is horizontal.

(d) Let x and y be functions of time t that are related by the equation $y^2 = 2 + xy$. At time $t = 5$, the value of y is 3 and $\frac{dy}{dt} = 6$. Find the value of $\frac{dx}{dt}$ at time $t = 5$.

2004 AB 4 BC 4

Consider the curve given by $x^2 + 4y^2 = 7 + 3xy$.

(a) Show that $\frac{dy}{dx} = \frac{3y-2x}{8y-3x}$.

(b) Show that there is a point *P* with *x*-coordinate 3 at which the line tangent to the curve at *P* is horizontal. Find the *y*-coordinate of *P*.

(c) Find the value of $\frac{d^2y}{dx^2}$ at the point *P* found in part (b). Does the curve have a local maximum, a local minimum, or neither at the point *P*? Justify your answer.

2001 AB 6

The function *f* is differentiable for all real numbers. The point $\left(3, \frac{1}{4}\right)$ is on the graph of $y = f(x)$, and the slope at each point (x, y) on the graph is given by $\frac{dy}{dx} = y^2(6 - 2x)$.

(a) Find $\frac{d^2y}{dx^2}$ and evaluate it at the point $\left(3, \frac{1}{4}\right)$.

(b) Find $y = f(x)$ by solving the differential equation $\frac{dy}{dx} = y^2(6 - 2x)$ with the initial condition $f(3) = \frac{1}{4}$.

Name: _____ **Class:** _____ **Date:** _____ **ID: A**

Implicit Differentiation

Multiple Choice
Identify the choice that best completes the statement or answers the question.

____ 1. The slope of the line tangent to the curve $y^2 + \left(xy + 1\right)^3 = 0$ at (2, -1) is

a. $-\frac{3}{2}$
b. $-\frac{3}{4}$
c. 0
d. $\frac{3}{4}$
e. $\frac{3}{2}$

____ 2. If $\frac{dy}{dx} = \sqrt{1 - y^2}$, then $\frac{d^2y}{dx^2} =$

a. $-2y$
b. $-y$
c. $\frac{-y}{\sqrt{1 - y^2}}$
d. y
e. $\frac{1}{2}$

____ 3. If $x^2 + y^2 = 25$, what is the value of $\frac{d^2y}{dx^2}$ at the point (4, 3) ?

a. $-\frac{25}{27}$
b. $-\frac{7}{27}$
c. $\frac{7}{27}$
d. $\frac{3}{4}$
e. $\frac{25}{27}$

____ 4. A railroad track and a road cross at right angles. An observer stands on the road 70 meters south of the crossing and watches an eastbound train traveling at 60 meters per second. At how many meters per second is the train moving away from the observer 4 seconds after it passes through the intersection?

a. 57.60
b. 57.88
c. 59.20
d. 60.00
e. 67.40

____ 5. If $y = xy + x^2 + 1$, then when x = -1, $\frac{dy}{dx}$ is

a. $\frac{1}{2}$
b. $-\frac{1}{2}$
c. -1
d. -2
e. nonexistent